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Some aspects of bow resonances - conditions for spectral influence on the bowed string

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Abstract

The dynamics of the frictional force between the bow hair and the string causes the bow to oscillate in its longitudinal direction during the bow stroke. Under certain conditions, such oscillations in the bow - enhanced by the bow resonances - can be observed in the spectrum of the frictional force between bow and string, and also in the velocity spectrum at the bridge. The present study discusses the necessary conditions for such an interaction to be noticeable. Further, the influence of bridge resonances on the spectrum of the frictional force is examined. The analysis is based on computer simulations in combination with observations on real bows and strings, when played by a bowing machine.

Introduction

The experience of every professional string player is that "different bows produce different sound from the instrument." Also, "some bows are easier to play than others". The explanation to such statements may be found in small fluctuations in the velocity of the bow hair - to some degree due to the resonances of the bow - which are superimposed on the steady bow velocity supplied by the player. As this study will show, these fluctuations can be observed (under certain conditions at a substantial amplitude), but there is no simple relation between their magnitudes and the impact on the force spectrum at the bridge. The transfer function is composed of several elements, some of which will be examined in the following.

Overview of elements in the bow - bridge transfer link

Bow resonances

The presence of oscillations in the bow hair and their connection with the resonances of the bow have been discussed by Schumacher (1975) and others (Cremer, 1984; Askenfelt, 1993, 1995). In playing, the bow resonances are excited by changes in the frictional force which occur during the entire period, but most significantly during the sticking part, and, in particular, at the transitions between stick and slip (Fig. 1). The bow resonances will couple to the string resonances and transmit, reflect, and/or absorb some part of the arriving energy, depending on the admittance ratio between bow and string.

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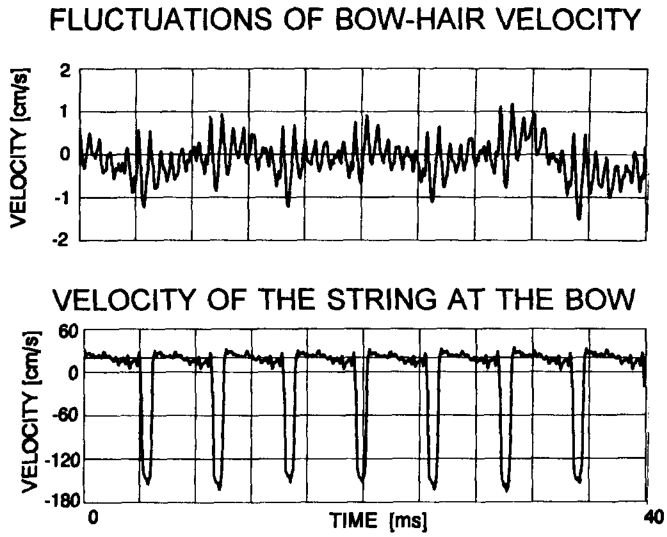

Fig. 1. Velocity in the bow hair (top) and string velocity (bottom) during a steady-state stroke by a bowing machine. The bow hair velocity is seen to fluctuate with amplitudes up to 1.5 cm/s. Frequencies lower than the fundamental of the string are present. Bow velocity 20 cm/s and bow force 800 mN.

Bow and string admittances

During the sticking interval, the frictional force works on three mechanical admittances: (1) the bow admittance; (2) the transversal string admittance; and (3) the torsional string admittance. The string admittances here include the reflections from the string terminations. All three admittances are frequency dependent and the admittance ratios vary grossly with frequency. The transverse admittance of the string is at most frequencies much higher than the torsional admittance, which in turn is higher than the admittance of the bow (cf. Figs. 3 and 4).

The transversal (or torsional) point admittance of a string with reflecting terminations can be given in a simple equation, provided that all losses during propagation are taken into account by the reflection functions (Guettler, 1994):

$$Y(x, j\omega) = \frac{1}{2Z}\left[\frac{1}{1 + R_{BR}(j\omega)\, e^{-j\omega 2X/C}} + \frac{1}{1 + R_{NUT}(j\omega)\, e^{-j\omega 2(L-X)/C}} - 1\right]^{-1} \qquad \text{Eq. (1)}$$

where

$Y(x, j\omega)$	string point admittance
Z	characteristic wave impedance of the string
x	distance from bridge to bowing point
L	string length
C	propagation velocity
$R_{BR}(j\omega)$	reflection function at the bridge
$R_{NUT}(j\omega)$	reflection function at the nut

$Y(x, j\omega)$ and all variables except x and L, refer to either the transversal or the torsional case.

String - bridge transfer function.

The transfer function from an arbitrary point on the string to the bridge (under the same conditions as for Eq.1), is given by Guettler (1994):

$$\frac{v_{BR}(j\omega)}{v_X(j\omega)} = \frac{[1 + R_{BR}(j\omega)]\, e^{-j\omega X/C}}{1 + R_{BR}(j\omega)\, e^{-j\omega 2X/C}} \qquad \text{Eq. (2)}$$

where

$v_{BR}(j\omega)$ velocity of the bridge

$v_X(j\omega)$ velocity of the string at the contact point with the bow

The transfer function of the velocity of the string to the bridge is strongly dependent on the bridge admittance. Even more important, this function shows a periodic pattern due to the circular functions in the denominator, with peaks close to nf_o / β ("the nodal frequencies"), f_o being the fundamental frequency and $\beta = x/L$. At these frequencies, the string has a low input admittance as seen by the bow, (in fact, of the same order as the admittance of the bow itself), and the transmission is strongly favoured. Consequently, if the relatively modest resonant fluctuations in the velocity of the bow hair are to have an impact on the bridge spectrum, the chances are indeed greatest near these nodal frequencies.

Spectrum of the frictional force

During the sticking interval, the bow (moving with the constant velocity V_B) forces the string to follow in the same direction. During the same interval, periodic reflections of waves on the string will be present, causing variations in the frictional force. The most important set of reflections are the transverse waves travelling "ping-pong" between the bow and the bridge. These reflected waves are initially excited at the release and capture of the string at the bow (Schumacher, 1979), and repeat with a frequency equal (or close) to f_o / β. This is seen in the transverse point impedance, which shows a peak at this frequency, making the string harder to excite. As a consequence, a significant peak will also occur in the spectrum of the frictional force. The height of this peak is, however, reduced if *either* the torsional string impedance, the bridge impedance, or the bow-hair impedance shows a valley in the same region. Simulations indicate that the most dominant maxima in the frictional force spectrum usually are found at f_o and near nf_o / β.

The bow will respond to the frictional force according to its admittance. This follows from the fact that the fluctuations in bow velocity at the contact point with the string in principle may be regarded as a convolution between the impulse response of the bow, measured in the longitudinal direction of the bow hair, and the derivative of the frictional force. This fact underlines the importance of the details of the frictional force spectrum.

Spectral magnitudes of string velocity at the bowing point

The measured spectral magnitudes of the bow-hair velocity are generally low compared to the magnitudes of the string velocity, as measured at the bowing point (Fig. 2). For normal values of β, the amplitude of the fundamental component in the string spectrum will be as high as $2V_B$, or twice the steady-state bow velocity. The difference in velocity between the string and bow components reduces the possibilities of an impact on timbre due to fluctuations in bow velocity. Around the node frequencies, however, the string velocity spectrum has pronounced dips, and at the same frequencies the frictional force spectrum shows peaks, so exactly in these regions there might be chances for an influence of the bow resonances on the string motion.

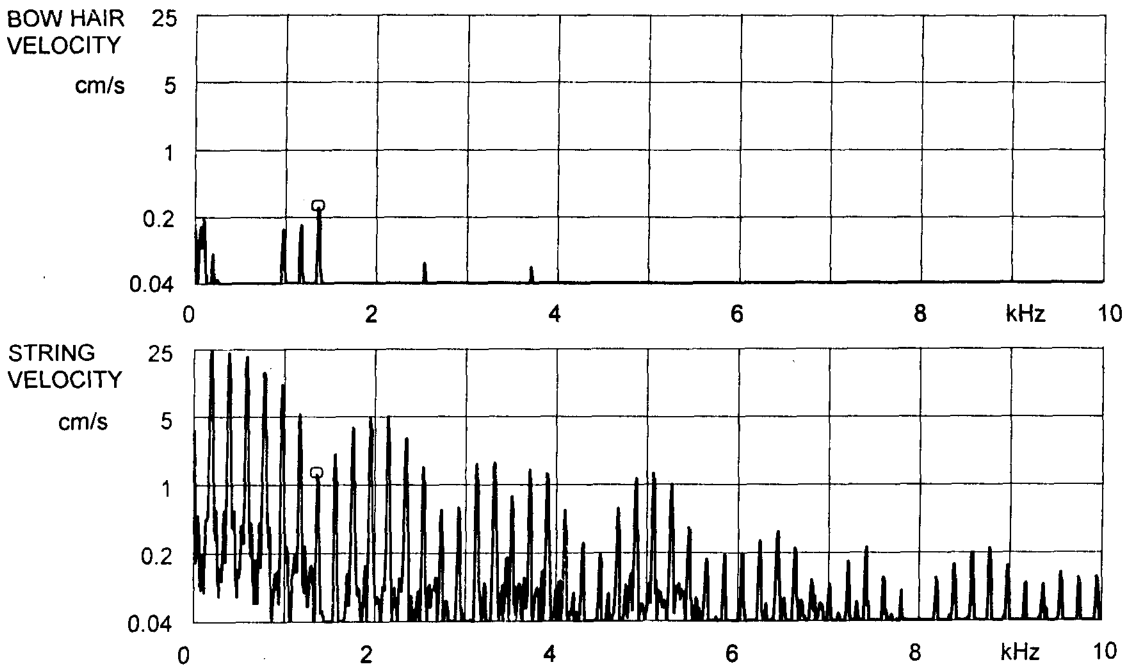

Fig. 2. Measurements of the velocity at the middle of the bow hair (top), and the string velocity at the contact point with the bow (bottom) during a stroke by a bowing machine. Relative bowing position β ≈ 1/7. Lowest "node frequency" indicated by circles.

Time windows for static and sliding friction

During the sliding interval, the string is more or less decoupled from the bow hair, and the transmission of bow resonances is much reduced. During the static interval, however, waves have been emitted on the string, both towards the bridge and towards the nut. As a consequence, waves will arrive at (and mostly pass) the bow after having been reflected at the nut at a time $0.5(1\text{-}\beta)/f_o$ earlier (during the sliding interval).

On a torsion free string, the spectral magnitude of the longitudinal bow-hair velocity (near the contact point with the string) will for most harmonic frequencies nf_o, be nearly equal to the spectral magnitude of the transverse string motion. The exceptions are found at the node frequencies, at which the transmission is minimal, but sidelobe frequencies can reach high magnitudes due to an "on/off switching" effect between the two time windows (stick and slip, respectively). This implies that any frequency near

$nf_0 \approx f_0(m + 0.5)/\beta$ will be effectively transferred between the bow and the string ($0 \le m < n$), while any frequency near $nf_0 = m/\beta$ ($0 < m < n$) will be suppressed. The frequencies $(n-1)f_0$ and $(n+1)f_0$ will, however, reach high values due to the switching/sidelobe effect. When torsion is present as well, an exchange between rotational and translational transmission takes place, according to their relative transfer characteristics and point admittances.

Other elements that may cause audible changes of timbre

(1) The violin bow has major resonances below the frequency range of the instrument (see e.g. Askenfelt, 1995). In this frequency range, the violin body is a poor radiator. There is nonetheless a possibility that these low frequencies are perceived by the ear through amplitude modulation of the partial frequencies in the violin spectrum. In particular, during the attack transients where the frictional force fluctuates vividly, also at frequencies lower than the fundamental, a real possibility for a detection of the low bow resonances could be expected.

(2) As described by McIntyre et al. (1981), the length of the fundamental periods in violin playing fluctuate somewhat. Simulations show that both string torsion and bow resonances could cause such fluctuations, as both may have mode frequencies which can interfere with the transversal string modes. Apart from a "phaser effect" of such a combination, also a widening of the peaks in the spectrum may increase the strength of (resonant) "near-harmonic" frequencies, in a similar way as a vibrato does.

Comparison between string and bow admittances

Transverse and torsional point admittances

Fig. 3 shows transverse and torsional point admittances of a "heavy" violin G-string excited at $\beta = 0.08 = 1/12.5$. These admittances were obtained through computer modelling by exciting a string with white-noise and averaging a series of FFT's of the string velocity at the contact point $v_x(\omega,x)$ divided by the frictional force of a sticking bow. The characteristic string impedances were set to 370 g/s for transversal waves, and 925 g/s for torsional waves, respectively. Pickering (1985) reports values from 274 to 386 g/s for transverse wave impedances of violin G-strings, while the impedances for higher strings are generally lower. The ratio between the torsional and transverse wave propagation velocities (the relative velocity C_{TOR}/C_{TRV}), was set to 4.8 in the simulations, and the Q-values to $245 < Q_{TRV} < 525$ and $17 < Q_{TOR} < 31$ within a bandwidth of 10 kHz. These simulation parameters were used in all the simulation examples which follow.

In passing, it is interesting to notice that when comparing simulations of a system with transverse modes of high Q-values with a torsional system of low Q-values, both types being present in the violin string, the point-admittance curve of the latter converges much faster (toward half the characteristic wave admittance) at the high-frequency end. This implies greater torsional influence on the string surface admittance ($Y_{TRV} + Y_{TOR}$) for the low harmonics.

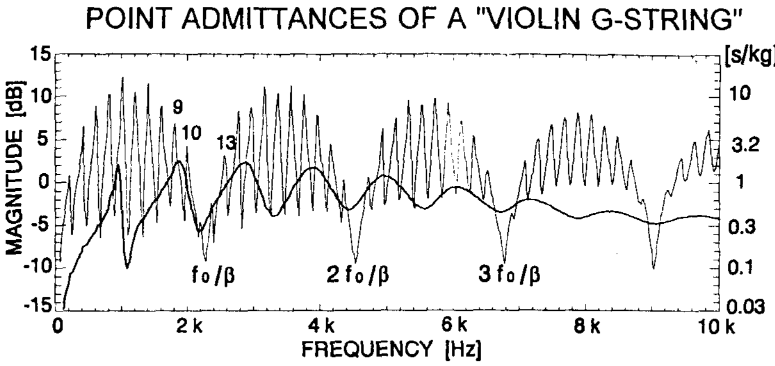

Fig. 3. Transversal and torsional point admittances of a high tension violin G-string (curves obtained through computer simulations, see text). At most mode frequencies, the transverse admittance is significantly higher than the torsional.

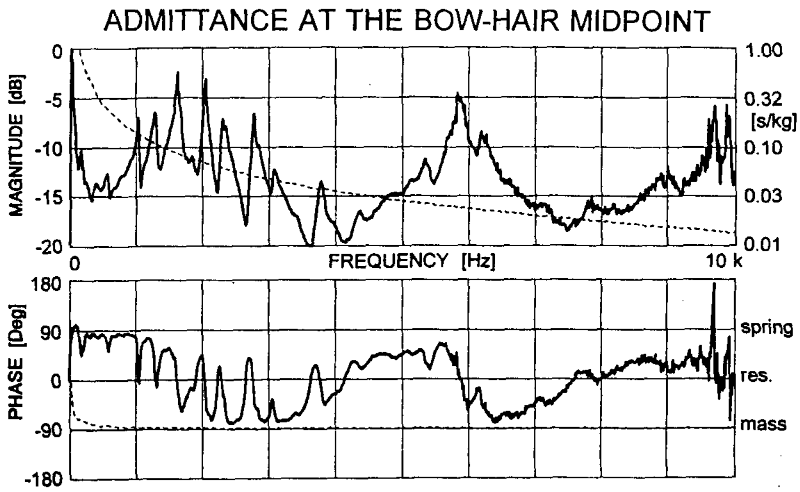

Fig. 4. Admittance of a violin bow measured with an accelerometer in the bow hair. The admittance of the accelerometer (dashed line) has been compensated for. The frequency range above 5 kHz should be interpreted with some caution as it may contain spurious information.

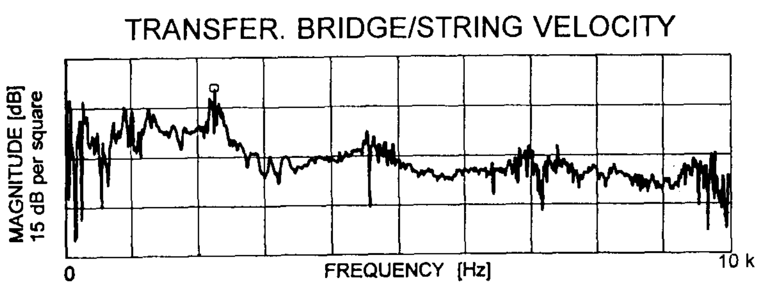

Fig. 5. Transfer function for a violin G-string calculated as bridge velocity divided by velocity of the string at the bowing point. The frequency $C_{TRV}/2X \approx 2.25$ *kHz, corresponding to the reflections travelling ping-pong between the bow and the bridge, is encircled.*

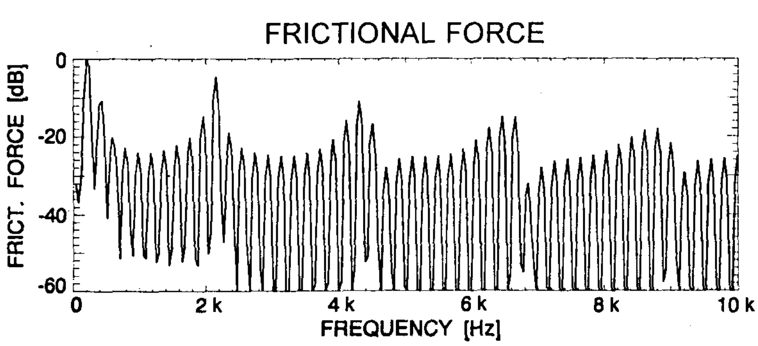

Fig. 6. Simulated frictional-force spectrum of an open G-string bowed at $\beta = 1/11.5$ *(the same* β *as in Figs. 3 and 5). High amplitudes occur at* f_0 *and at integer multiples of* f_0/β*.*

As can be seen from the comparison of the transversal and torsional string admittances in Fig. 3, the transverse admittance is by far the highest of the two at most mode frequencies. This implies that only little transversal kinetic energy will be transformed into torsional energy at these harmonics. However, in a narrow region around the 11th and 12th harmonic (near f_0 / β) the difference in admittances is small. If the relative velocity C_{TOR} / C_{TRV} had been set to 4.0, a substantial amount of the transverse kinetic energy would instead have been transformed into torsional energy at these frequencies, because then the torsional admittance would have had its third-mode peak exactly at $12f_0$, and the amplitudes of these harmonics would have dropped. On the other hand, the 9th and 10th harmonic would have gained from this change, then being located in valley of the torsional admittance curve. When comparing these simulations with two different velocity ratios, a very noticeable difference (of the order of 6 - 8 dB or more) in the bridge spectrum can be observed at some frequencies. Simulations like these suggest that the spectral profile is more sensitive to the transverse - torsional propagation velocity ratio C_{TOR} / C_{TRV}, than to the characteristic impedance ratio.

Schumacher (1979) has reported that, formally, the bow admittance can be viewed in the same way as the torsional string modes. Consequently, the bow possesses a potential of influencing the string spectrum to a degree similar to that of the string itself, provided the bow admittance is high enough compared to the transversal point admittance of the string.

Bow admittance

In order to measure the admittance of the bow as seen by the string, a miniature accelerometer was fastened in the bundle of hair (with all hairs in contact), and hit with a miniature force hammer. Measurements were taken both in the longitudinal and transverse direction of the bow hair, and at different positions along the bow (tip - middle - frog). Fig. 4 shows the admittance obtained when measuring the longitudinal admittance of the bow hairs at the midpoint of the bow. The measurement is influenced by the accelerometer, the mass of which (1.1 g) is of the same magnitude as that of the bow hair (a complete bundle of bow hairs weighs between 4 and 5 g). A compensation is therefore necessary and has been applied to the admittance curve in Fig. 4. As the compensation is relatively large, even a small uncertainty in the compensation admittance will have a rather large influence on the result, and in particular, on the phase information. The measurements above about 5 kHz should therefore be interpreted with some caution. Concerning the individual bow characteristics, the major resonances of the bow fall well below this frequency. For further details on the measurements of bow admittance, see Askenfelt (1995).

Different measuring positions on the bow ("at the frog" and "at the tip") give slightly different amplitudes of the resonance peaks, particularly above 2 kHz. Some of these reach slightly higher values than in Fig. 4. In general, however, the admittance of the bow as "seen by the string," is much lower than the string admittances of the transversal string modes. For higher tuned strings with higher characteristic admittances (up to 3 - 4 dB), the admittance gap between bow and string is even greater, and the chances for an influence on the string spectrum due to the bow resonances are reduced accordingly. For other bowed instruments operating in lower frequency ranges, like the cello and double bass, the bow and string admittances may be closer matched. However, the question of which bow is the superior one - the resonant or the non-resonant - remains to be answered.

The transfer function

Fig. 5 shows the measured transfer function of a violin G-string calculated as the velocity of the bridge divided by string velocity at the bow. The bowing point was equal to L/11.5 as in Fig. 3. The characteristic periodic peaks in the transfer function are clearly visible (cf. Eq. 2). The measured transfer function shows typical indications on string stiffness (expanding intervals between resonance peaks towards high frequencies), and energy dissipation (lowered transfer ratio for higher frequencies).

The transfer function was measured in a rather unsophisticated way, just for the demonstration of the periodic "sinusoidal" pattern. With the bow mounted in the bowing machine (thus maintaining a constant bowing position) the bow was manually "scratched" over the string, exciting only a weak hiss. The string was efficiently damped by pads of foam rubber at the nut side. The transfer function in Fig. 5 is the average of 10 such registrations.

The frictional force

Fig. 6 shows a simulation of the frictional force when the string model is bowed with a constant velocity and a constant (high) bow force. The force components at f_0 and f_0/β are seen to dominate. It is interesting to note that the frictional force spectrum is influenced by the bridge impedance to the same extent as the string point admittance is influenced by the bridge reflection function. This follows from the fact that at frequencies near $nf_0/\beta \approx nC_{TRV}/(2\beta L)$, where $\cos(2\omega X/C_{TRV}) = 1$, the expression $1 + R_{BR}(j\omega)$ in Eq. 1 can be substituted with expressions for impedance; $1+ R_{BR}(j\omega) = 2Z/(Z+Z_{BR}(j\omega))$, where $Z_{BR}(j\omega)$ is the impedance of the unstrung bridge. The effect of bridge impedance on the transverse string point admittance can then more easily be seen. At the node frequencies, the string point admittance increases with increasing bridge admittance. Between these frequencies, the influence of the bridge impedance is reduced, and varies according to the periodic "sinusoidal" pattern.

Influence observed through simulations

In order to investigate to what degree peaks in the bow admittances like those shown in Fig. 4 would modify the output spectrum at the bridge, a series of simulations were performed. For each simulation, a single bow resonance was programmed by use of a convolution function. The bow admittance used in one of these simulations is shown in Fig. 7, with an admittance amplitude of 0.5 s/kg (-3 dB). The amplitude of the admittance peak was kept constant in all simulations. The influence on the bridge spectrum for different frequencies of this single bow resonance (f_{RES}) is shown in Fig. 8. The resulting spectrum is compared to a simulation with a purely resistive bow admittance of 0.04 s/kg (-14 dB). The simulations show that for f_{RES}/f_0 = 5.0, 13.0, 14.0, and 15.0, the influence is hardly observable, the admittance of the bow being small compared to the point admittances of the string at f_{RES}. However, when f_{RES} is close to a node frequency, significant spectral changes are observable. In the present example this occurs when f_{RES}/f_0 = 11.0 and 23.0, as our β was chosen as 1/11.5.

SIMULATED BOW/HAIR ADMITTANCE

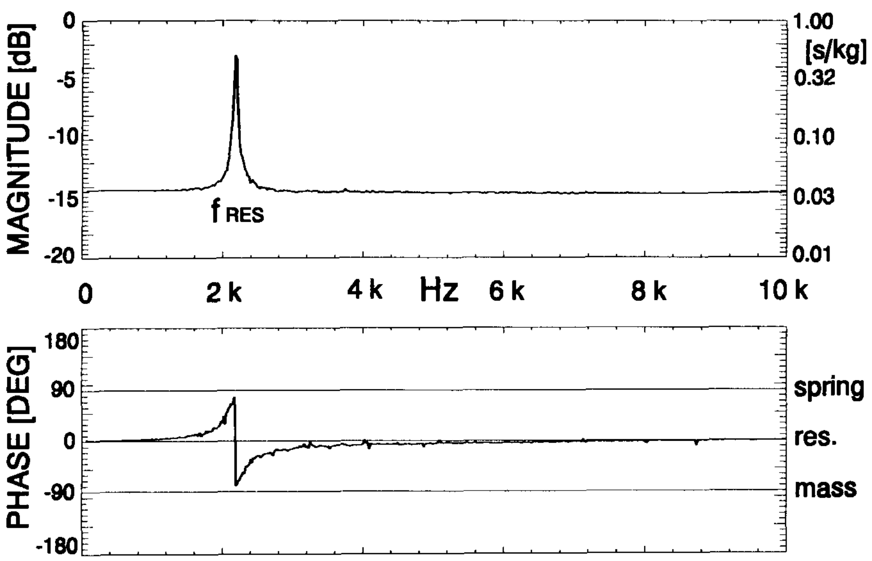

Fig. 7. Simulated admittance of a bow with only one resonance. In Fig. 8, this type of bow is used to study the influence on the output spectrum, using a convolution function. The magnitude of the admittance peak is kept constant at 0.5 s/kg (-3 dB) as the resonance frequency is varied, while the Q-value is set to $f_{RES}/52$. *For the resonance in the figure Q equals 42.*

Conclusions

The simulations and measurements described above give strong indications of that under certain conditions an influence on the string and bridge spectra due to the resonances of the bow is possible. In particular, the calculations and measurements of the admittances of the string and the bow have shown that in certain frequency regions, partly dependent on the bow position, the transverse point admittance of the string is low enough to be comparable to the peak admittances of the bow at the resonances. Further, simulations suggest that a very noticeable influence on the output spectrum may occur as a result of realistically scaled bow-hair resonances, provided that these happen to be located near a "node frequency." However, in experiments with real bows and strings, there is currently no clear evidence of that the bow resonances are excited sufficiently strongly, as to have a significant influence on the output spectrum, even though considerable resonant fluctuations in the bow hair velocity were observed. The work on a better understanding of the bow and its influence on the string motion and violin timbre will be continued.

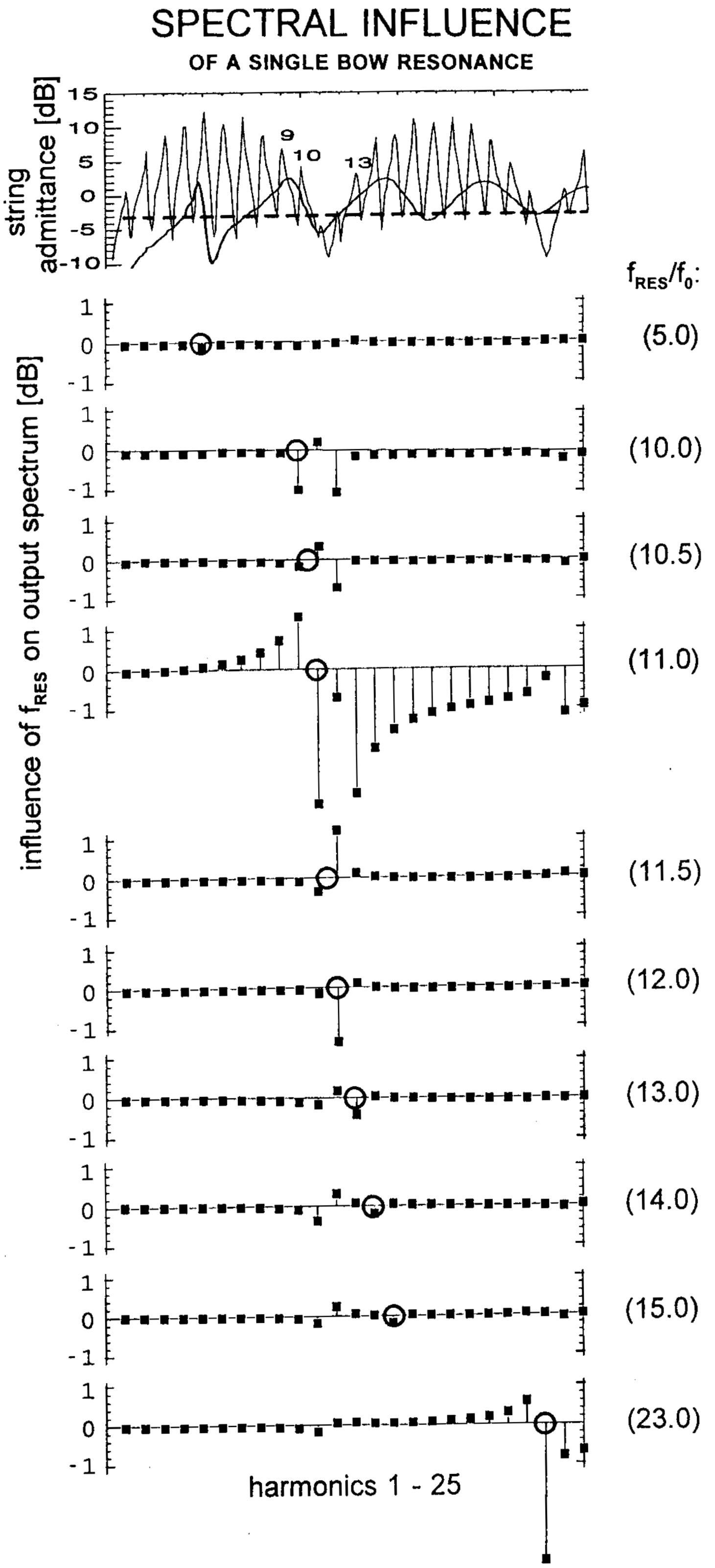

Fig. 8. Simulated spectral influence of a bow resonance on the output bridge spectrum. The upper graph shows the same transversal and torsional point admittances as in Fig. 3. The dashed, horizontal line indicates the (peak) level of the bow admittance at its resonance frequency f_{RES}. *Below, plots of the difference between the output spectrum of a string bowed with a resonant and a non-resonant bow are shown for 10 different frequencies of the bow resonance. The relative frequency* f_{RES}/f_0 *is marked with a circle in each plot. Notice that when the resonance frequency of the bow exactly matches the frequency of a string harmonic, this harmonic will be lowered in the bridge output spectrum. All simulations were run with the same bow velocity and bow force. The durations of the resulting stick-slip intervals turned out to be identical in all simulations.*

Acknowledgements

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